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SPEED AND ATTENUATION OF SOUND IN GAS – VAPOR – LIQUID SYSTEMS. ROLE OF HEAT AND MASS EXCHANGE

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Several theoretical and experimental papers have been devoted to the study of the propagation of acoustic excitations in one and two-component, two-phase media of the gas suspension type [1-13]. The propagation of small-amplitude acoustic excitations in a mixture of vapor or gas with liquid drops was considered in [1-3]. Excitations of finite amplitude were considered in [4, 5]. The dispersion and absorption of weak sound waves was studied in [6-12] for a mixture of an inert gas with liquid drops and water vapor. The propagation of finite-amplitude excitations in fog was analyzed in [13]. The effect of the unsteady interaction of the phases on the propagation of high-frequency excitations was studied in [2, 3] for single-component mixtures of vapor and liquid drops. In the present paper we study the dispersion and attenuation of sound in one and two-component gas–liquid mixtures.

1. Basic Equations of Motion and Equations of State. We consider monodispersed mixtures and assume acoustic homogeneity. In order to study the phenomena, we use the model of a two-velocity and three-temperature continuum [14]. We consider the linearized equations of motion in the plane, one-dimensional case in the presence of phase transitions. In a coordinate system in which the unperturbed mixture is at rest, the conservation equations of mass and momentum for the phases are

$$\begin{gathered}\frac{\partial\rho_1'}{\partial t}+\rho_{10}\frac{\partial v_1'}{\partial x}=-nj_{V\Sigma},\ \frac{\partial\rho_V'}{\partial t}+\rho_{V0}\frac{\partial v_1'}{\partial x}=-nj_{V\Sigma},\ \frac{\partial\rho_2'}{\partial t}+\rho_{20}\frac{\partial v_2'}{\partial x}=nj_\Sigma,\\ \rho_{10}\frac{\partial v_1'}{\partial t}+\frac{\partial p_1'}{\partial x}+nf=0,\ \rho_{20}\frac{\partial v_2'}{\partial t}=nf,\\ \rho_{10}=\alpha_{10}\rho_{10}^0,\ \rho_{20}=\alpha_{20}\rho_{20}^0,\ \alpha_{10}+\alpha_{20}=1,\ \alpha_{20}=\frac{4}{3}\pi a_0^3n,\\ \rho_1=\rho_V+\rho_G,\quad p_1=p_V+p_G,\end{gathered}\tag{1.1}$$

where ρ and ρ^0 are the reduced and true densities; v and p are the velocity and pressure; α is the volume content; n is the number of particles per unit volume; f is the force on an individual liquid drop due to the carrier phase; $j_{V\Sigma}$ is the diffusive flux of vapor to the surface Σ of a drop; j_Σ is the rate of condensation onto the surface of an individual drop. Here and below the subscripts 1 and 2 refer to the gaseous phase and the suspended phase, V and G refer to the vapor and gas components of the carrier phase, and the primes denote small perturbations, while the subscript 0 denotes the initial unperturbed state.

The equations governing the supply of heat to the gaseous phase, to the drops, and to the surface of an individual drop can be written as

$$\begin{gathered}\rho_{V0}\frac{\partial i_V'}{\partial t}+\rho_{G0}\frac{\partial i_G'}{\partial t}=\alpha_{10}\frac{\partial p_1'}{\partial t}-nq_{1\Sigma},\ \rho_{20}\frac{\partial u_2'}{\partial t}=-nq_{2\Sigma},\\ q_{1\Sigma}+q_{2\Sigma}=-j_\Sigma l,\end{gathered}\tag{1.2}$$

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where i, u, l are the specific enthalpy, internal energy, and heat of vaporization; $q_{j\Sigma}$ is the intensity of heat exchange between the j-th phase and the surface of a drop (j = 1, 2). We will assume that the thermal and physical parameters of the gaseous phase (consisting of vapor and gas) can be determined from the corresponding parameters of the vapor and the gas:

$$
\begin{gathered}
R_1 = k_V R_V + k_G R_G, \quad c_{p1} = k_V c_{pV} + k_G c_{pG}, \\
\lambda_1 = k_V \lambda_V + k_G \lambda_G, \quad \mu_1 = k_V \mu_V + k_G \mu_G.
\end{gathered}
\tag{1.3}
$$

Here R, c_p, λ, and μ are, respectively, the universal gas constant, the heat capacity at constant pressure, the thermal conductivity, and the dynamical viscosity; k_V and k_G are the concentrations of vapor and gas in the carrier phase of the mixture:

$$
k_V = \rho_V/\rho_1, \; k_G = \rho_G/\rho_1, \; k_V + k_G = 1. \tag{1.4}
$$

It is assumed that the components of the gaseous phase are ideal gases.* Then the linearized equations of state of the vapor and the gas mixture as a whole can be written in the form:

$$
\begin{gathered}
\frac{p_V'}{p_{V0}} = \frac{\rho_V^{0\prime}}{\rho_{V0}^0} + \frac{T_1'}{T_0}, \quad i_V' = c_{pV} T_1', \\
\frac{p_1'}{p_{10}} = \frac{\rho_1^{0\prime}}{\rho_{10}^0} + \frac{T_1'}{T_0} + \frac{R_1'}{R_{10}}
\end{gathered}
\tag{1.5}
$$

(T is the temperature). The equation of state of the incompressible dispersed phase is

$$
\rho_2^{0\prime} = 0, \quad u_2' = c_2 T_2', \quad u_2' = c_2 T_2' \tag{1.6}
$$

(c_2 is the specific heat of the condensed phase).

The Clausius–Clapeyron equation holds along the phase equilibrium curve [14]

$$
\frac{dT_S(p_V)}{dp_V} = \frac{T_S(p_V)\left(1 - \rho_V^0/\rho_2^0\right)}{l \rho_V^0} \tag{1.7}
$$

(the subscript S refers to parameters on the saturation curve).

We will consider solutions of the system of linear equations (1.1) through (1.7) in the form of traveling waves for the perturbation ψ':

$$
\begin{gathered}
\psi' \sim \exp i\,(K_* x - \omega t) = \exp(-K_{**} x) \exp i\,(Kx - \omega t) \\
(K_* = K + iK_{**}, \; C_p = \omega/K, \; C_g = d\omega/dK, \; \sigma = 2\pi K_{**}/K),
\end{gathered}
\tag{1.8}
$$

where i is the imaginary unit; K_* is the complex wave number; K_{**} is the linear attenuation factor; C_p and C_g are the phase and group velocities; σ is the attenuation decrement per wavelength.

2. Unsteady Effects Due to Interactions between the Phases. In the presence of a propagating sound wave, the thermodynamic equilibrium between the phases of the mixture is periodically disrupted in the regions of compression and expansion. Unsteady exchange of mass, momentum, and energy can occur between the phases.

Analysis of the results of [2] shows that when $\alpha_{20} \ll 1$, $\rho_2^0 \gg \rho_1^0$ (a dilute suspension at moderate pressure) the most important forces acting on an individual particle of the dispersed phase are the Stokes and Bassi forces. For an excitation of the type (1.8), the total force f can be written as

$$
\begin{gathered}
\frac{nf}{\rho_{20}} = \frac{v_1' - v_2'}{\tau_v^*}, \quad \tau_v^* = \tau_v \left[1 + \frac{1 - i}{\sqrt{2}} (\omega \tau_{\mu 1})^{1/2}\right]^{-1}, \\
\tau_v = \frac{2}{9} \frac{\rho_{20}^0 a^2}{\mu_1} = \frac{2}{9} \frac{\rho_{20}^0}{\rho_{10}^0} \tau_{\mu 1}, \quad \tau_{\mu 1} = \frac{\rho_{10}^0 a^2}{\mu_1} \left(\tau_v \gg \tau_{\mu 1}, \; \rho_{20}^0 \gg \rho_{10}^0\right).
\end{gathered}
\tag{2.1}
$$

Here $\tau_{\mu 1}$ is the characteristic time to establish a quasisteady velocity distribution in the gaseous phase; τ_V is the Stokes velocity relaxation time for the phases; τ_V^* is its complex analog. The difference between τ_V^* and

---

† We emphasize that when $R_V \neq R_G$ and when there are phase transitions in the mixture, the vapor and gas are not ideal gases in view of the dependence of R_1 on k_V, as given in (1.3).

the Stokes relaxation time becomes important for frequencies $\tau_{\mu 1}^{-1} < \omega \ll \omega_c$ ($\omega_c = 2\sqrt[3]{\alpha_{20}}\,\pi C_1/a$ is the characteristic oscillation frequency of an excitation whose wavelength is comparable to the distance between the inclusions, C is the speed of sound).

The Prandtl number of the gaseous phase satisfies Pr ~ 1, hence unsteady heat exchange effects appear for the same oscillation frequencies as unsteady effects associated with the interaction force between the phases. The fluxes $q_{j\Sigma}$ (j = 1, 2) depend on the frequency ω in the form [15]

$$\begin{gathered}
\frac{nq_{1\Sigma}}{\rho_{10}c_{p1}} = \frac{T_1' - T_\Sigma'}{\tau_{T1}^*},\ \frac{nq_{2\Sigma}}{\rho_{20}c_2} = \frac{T_2' - T_\Sigma'}{\tau_{T2}^*}, \\
\tau_{T1}^* = \frac{1}{3}\frac{\alpha_{10}}{\alpha_{20}}\tau_{\lambda 1}\eta_1(z_1),\ \tau_{T2}^* = \frac{1}{15}\tau_{\lambda 2}\eta_2(z_2), \\
\eta_1(z_1) = \frac{1}{1+z_1},\ \eta_2(z_2) = \frac{5\left[3z_2 - \left(3+z_2^2\right)\operatorname{th} z_2\right]}{z_2^2(\operatorname{th} z_2 - z_2)}, \\
z_j = \frac{1-i}{\sqrt{2}}(\omega\tau_{\lambda j})^{1/2},\ \tau_{\lambda j} = \frac{a^2}{\varkappa_j},\ \varkappa_j = \frac{\lambda_j}{\rho_{j0}^0 c_j},\quad j = 1, 2,
\end{gathered} \tag{2.2}$$

where τ_{Tj}^* is the complex temperature relaxation time in the j-th phase, and is determined by the characteristic time $\tau_{\lambda j}$ and by the frequency ω; $\varkappa$ is the thermal diffusivity.

The Schmidt number Sc ~ Pr, and therefore when the unsteady effects discussed above need to be taken into account, it is necessary to take into account unsteady mass exchange as well. According to [15], the ω-dependence of the diffusive flux $j_{V\Sigma}$ of vapor to the interfacial surface can be obtained from the solution of a spherically symmetric problem for the mass diffusion between an individual drop and the vapor–gas mixture in the presence of a weak monochromatic wave [6]:

$$\frac{nj_{V\Sigma}}{\alpha_{20}\rho_{10}^0} = \frac{1}{\tau_p^*}\frac{p_V' - p_{V\Sigma}'}{p_{10}}. \tag{2.3}$$

Here τ_p^* is the complex relaxation time for the partial pressure of the vapor, which is determined by the characteristic time τ_d and by ω:

$$\begin{gathered}
\tau_p^* = \frac{1}{3}\frac{R_V}{R_{10}}(1 - k_{V0})\tau_d\varphi(y), \\
\tau_d = \frac{a^2}{D_1},\quad \varphi(y) = \frac{1}{1+y}\quad \left(y = \frac{1-i}{\sqrt{2}}(\omega\tau_d)^{1/2},\ |\varphi(y)| \leqslant 1\right).
\end{gathered} \tag{2.4}$$

The time τ_p^* is close to its quasi-steady real part $\tau_p = (R_V/3R_{10})(1 - k_{V0})\tau_d$ for frequencies satisfying $\omega\tau_d < 10^{-2}$.

If the phase transition on the boundary between the phases proceeds out of equilibrium, then the vapor pressure $p_{V\Sigma}$ on the boundary will differ from the saturation pressure $p_{VS}(T_\Sigma)$ [or equivalently, the temperature T_Σ of the surface of the drop will differ from the saturation temperature $T_S(p_{V\Sigma})$]. The rate of nonequilibrium condensation on the surface can be written with the help of the Hertz–Knudsen–Langmuir formula [14]

$$\frac{nj_\Sigma}{\alpha_{20}\rho_{10}^0} = \frac{1}{\tau_\beta}\frac{p_{V\Sigma}' - p_{VS}'}{p_{10}},\quad \tau_\beta = \frac{1}{3}\sqrt{\frac{2\pi}{\gamma_V}}\frac{\gamma_1 C_V a}{\beta C_1^2}, \tag{2.5}$$

where τ_β is the characteristic time to equalize the partial pressures of the vapor on the interphase boundary and depends on the value of the coefficient of accomodation β; γ is the adiabatic index. From the mass balance condition on the surface of the drop we have

$$j_{V\Sigma} = j_\Sigma. \tag{2.6}$$

The system of equations (1.1) through (1.7), and (2.1) through (2.6) is a closed system and can be used to study the propagation of acoustic excitations in a mixture of an inert gas and liquid drops, plus vapor.

3. Estimates of the Pressure and Temperature Differences. From (2.3), (2.5), and (2.6), we find an equation relating the partial pressures of the vapor p_V, $p_{V\Sigma}$, p_{VS}:

$$\frac{p_V - p_{V\Sigma}}{\tau_p^*} = \frac{p_{V\Sigma} - p_{VS}}{\tau_\beta}. \tag{3.1}$$

We use (3.1) to compare the characteristic pressure differences $|p_V - p_{V\Sigma}|$ and $|p_{V\Sigma} - p_{VS}|$ which occur when an acoustic excitation propagates in the vapor–gas suspension. Using (2.4) and (2.5) we have

$$\frac{\tau_\beta}{|\tau_p^*|} = \frac{\sqrt{2\pi}}{\beta}\sqrt{\frac{\gamma_1 R_{10}}{R_V}}\left(\frac{L}{a}\right)\frac{(1 - k_{V0})}{|\varphi(y)|}$$

($L \sim D_1/C_1$ is the mean free path of a molecule in the gaseous phase). For atmospheric fog with $L \sim 10^{-8}$, $k_{V0} \ll 1$, and a drop size of $a \sim 10^{-6}$ m, assuming the usually accepted order of magnitude of the coefficient of accomodation $\beta \sim 10^{-2}$, we have $\tau_\beta / |\tau_p^*| \geq 1$ for all frequencies. Hence over the frequency range $0 < \omega \ll \omega_c$ the pressure difference $|p_{V\Sigma} - p_{VS}|$ is always greater than or equal to the pressure difference $|p_V - p_{V\Sigma}|$. Therefore the departure from equilibrium of the interphase boundary, which is related to the difference between p_{VS} and $p_{V\Sigma}$, should be significant for all frequencies. We note that if $\beta \sim 1$ departure from equilibrium will appear only at high frequencies, when $|\varphi(y)| \sim 10^{-2}$.

In order to compare the characteristic temperature differences $|T_1 - T_\Sigma|$, $|T_2 - T_\Sigma|$, $|T_\Sigma - T_S|$, we use another form of the Hertz–Knudsen–Langmuir relation [14]:

$$\frac{n j_\Sigma l}{\alpha_{20}\rho_{10}^0 c_{p1}} = \frac{T_S - T_\Sigma}{\tau_\Sigma}, \quad \tau_\Sigma = \frac{\left(1 - \rho_{V0}^0/\rho_{20}^0\right)}{3(\gamma_1 - 1)}\sqrt{\frac{2\pi}{\gamma_V}}\,\frac{a C_V C_1^2}{\beta l^2 k_{V0}}, \tag{3.2}$$

where τ_Σ is the temperature relaxation time on the interphase surface, and is related to the pressure relaxation time τ_β by

$$\tau_\Sigma = \frac{\left(1 - \rho_{V0}^0/\rho_{20}^0\right)}{(\gamma_1 - 1)}\,\frac{C_1^4}{\gamma_1 l^2 k_{V0}}\,\tau_\beta. \tag{3.3}$$

Using (2.2) and (3.2), the equation for the supply of heat to the surface of the drop [compare (1.2)] can be written in the form

$$\begin{gathered}\frac{T_1 - T_\Sigma}{\tau_{\Sigma 1}^*} = \frac{T_\Sigma - T_2}{\tau_{\Sigma 2}^*} + \frac{T_\Sigma - T_S}{\tau_\Sigma},\\ \tau_{\Sigma 1}^* = \left(\frac{\alpha_{20}}{\alpha_{10}}\right)\tau_{T1}^* = \frac{1}{3}\tau_{\lambda 1}\eta_1(z_1), \quad \tau_{\Sigma 2}^* = \frac{\rho_{10}^0 c_1}{\rho_{20}^0 c_2}\tau_{T2}^* = \frac{\rho_{10}^0 c_1 a^2}{15\lambda_2}\eta_2(z_2).\end{gathered} \tag{3.4}$$

The following estimate will normally be valid for the ratio of the absolute values of $\tau_{\Sigma 1}^*$ and $\tau_{\Sigma 2}^*$:

$$\frac{|\tau_{\Sigma 2}^*|}{|\tau_{\Sigma 1}^*|} = \frac{1}{5}\left(\frac{\lambda_1}{\lambda_2}\right)\frac{|\eta_2|}{|\eta_1|} \ll 1, \quad (|\eta_2| < |\eta_1|, \ \lambda_1 \ll \lambda_2). \tag{3.5}$$

The ratio of the characteristic times τ_Σ and $|\tau_{\Sigma 2}^*|$ in a vapor–gas–liquid mixture (unlike the case of a one-component vapor suspension studied in [3]) depends not only on the thermal and physical parameters of the phases, but also depends on the initial concentrations of the phases:

$$\frac{\tau_\Sigma}{|\tau_{\Sigma 2}^*|} = \frac{15}{\beta k_{V0}}\left(\frac{\lambda_2}{\lambda_1}\right)\left(\frac{C_V}{C_1}\right)\left(\frac{C_1^2}{l}\right)^2\left(\frac{L}{a}\right)\frac{1}{|\eta_2|}. \tag{3.6}$$

For atmospheric fog, which is a mixture of air with water vapor and droplets of dimension $a \sim 10^{-6}$ m, at normal pressure and temperature, when $\lambda_2/\lambda_1 \sim 20$, $l/C_1^2 \sim 25$, $L \sim 10^{-8}$ m, and $k_{V0} \sim 10^{-2}$, the ratio $\tau_\Sigma/|\tau_{\Sigma 2}^*| \sim 10^{-1}/\beta(\eta_2)$. Therefore when $\beta \sim 10^{-2}$ we have $\tau_\Sigma/|\tau_{\Sigma 2}^*| \gg 1$ for any frequency. Thus in a two-component, two-phase mixture the following inequality, obtained from (3.4), (3.5), and (3.6), holds:

$$|T_2 - T_\Sigma| \ll |T_\Sigma - T_S| < |T_1 - T_\Sigma|.$$

Hence we concluded that for atmospheric fog (a mixture of air with vapor and droplets of water) the nonuniformity of temperature inside a droplet (the difference between T_2 and T_Σ) is usually small compared to the temperature nonuniformity in the gaseous phase (the difference between T_1 and T_Σ) and in contrast to the case of a one-component vapor suspension, it can be small in comparison to the departure from equilibrium of the interphase surface (the difference between T_Σ and T_S).

4. Dispersion Relation. We use the following dimensionless parameters for the composition of the mixture and the physical properties of the phases:

$$m=\frac{\rho_{20}}{\rho_{10}},\quad m^0=\frac{\rho^0_{20}}{\rho^0_{10}},\quad r=\frac{\rho^0_{10}}{\rho^0_{20}},\quad \bar{c}_1=\frac{c_{p1}}{\gamma_1 R_{10}}=\frac{1}{(\gamma_1-1)},\quad c_2=\frac{c_2}{\gamma_1 R_{10}},$$
$$\bar{l}=\frac{l}{C_1^2},\quad b=\frac{R_V}{R_{10}},\quad h=1-r+(1-k_{V0})\left(\frac{R_V-R_G}{R_{10}}\right).$$

From the condition that there exists a nontrivial solution of the system of linear equations [(1.1) through (1.7) and (2.1) through (2.6)] of the form (1.8), one can obtain the dispersion relation between the wavenumber and the excitation frequency:

$$(C_1K_*/\omega)^2=V(\omega)\,D(\omega),$$
$$V(\omega)=1+m\frac{F_1-F_2}{F_3+mF_2},\quad D(\omega)=1+m\frac{\gamma_1G_1+(\gamma_1-1)\,G_2}{G_3-mG_1}. \tag{4.1}$$

Here $V(\omega)$ and $D(\omega)$ are complex functions describing the dispersion and dissipative effects due to exchange of momentum between the phases and heat and mass exchange, respectively. When there are no particles ($m = 0$) we have $V(\omega) = D(\omega) = 1$, i.e., dispersion and absorption are absent in a gas without particles. The functions $V(\omega)$ and $D(\omega)$ depend on frequency and on the thermal and physical parameters of the phases through the functions F_j, G_j ($j = 1, 3$), and W_i ($i = 1, 4$):

$$F_1=1-c,\quad F_2=r\,[1-r\,(i\omega\tau^*_v)],\quad F_3=1-(i\omega\tau^*_v),$$
$$G_1=bk_{V0}(hW_3-\bar{l}W_2),$$
$$G_2=[1-(r+h)\,k_{V0}]\,bm\bar{c}_2+\gamma_1\frac{\bar{l}k_{V0}}{(1-rk_{V0})}(\bar{l}-h\bar{c}_1)\,W_2-m^0\bar{c}_2W_4,$$
$$G_3=m\left[(1-rk_{V0})\,bW_3-bk_{V0}\bar{l}W_2+\gamma_1\frac{\bar{l}^2k_{V0}}{(1-rk_{V0})}W_1W_2\right]-m^0W_3W_4, \tag{4.2}$$
$$W_1=1-(i\omega\tau^*_{T1}),\quad W_2=1-(i\omega\tau^*_{T2}),$$
$$W_3=m\bar{c}_2W_1+\bar{c}_1W_2,\quad W_4=i\omega\,(\tau^*_p+\tau_\beta).$$

In the limit $k_{V0} \to 1$, the dispersion relation (4.1), (4.2) reduces to that of [2], corresponding to the case of a single-component mixture of vapor and liquid drops.* The dispersion relation for a mixture of gas and particles in the absence of phase transitions [3] can be obtained from (4.1) and (4.2) by taking the limit $k_{V0} \to 0$.

It follows from (4.1), (4.2), and the estimates (3.5) and (3.6) that departure from equilibrium of the phase transition begins to become important for frequencies such that

$$W\,|\eta_2|\leqslant 1,\quad W=\frac{\beta k_{V0}}{15}\left(\frac{\lambda_1}{\lambda_2}\right)\left(\frac{R_{10}}{R_V}\right)\left(\frac{l}{C_1^2}\right)^2\left(\frac{a}{L}\right). \tag{4.3}$$

The characteristic oscillation frequencies satisfying the condition (4.3) on $|\eta_2|$ can be estimated by using the asymptotic formulas of [15]. Then for low frequencies ($\omega\tau_{\lambda 2} < 10$), when $|\eta_2| \sim 1$, the condition (4.3) takes the form $W \leq 1$. For high frequencies ($\omega\tau_{\lambda 2} > 10^3$) we have the asymptotic form $|\eta_2| \sim 10(\omega\tau_{\lambda 2})^{-1/2}$ and then (4.3) gives $(\omega\tau_{\lambda 2})^{1/2} \geqslant 10W$.

Hence, the processes of evaporation and condensation in two-component mixtures ($k_{V0} < 1$) will depart from equilibrium for lower frequencies than in the case of one-component mixtures ($k_{V0} = 1$). We note that if we formally take the limit $\beta \to \infty$ (τ_β, $\tau_\Sigma \to 0$) one can obtain from (4.1) and (4.2) a simpler dispersion relation, which, according to (4.3), is applicable to the propagation of excitations with frequencies for which $W\,|\eta_2| > 1$, i.e., mass exchange between the phases is a quasiequilibrium process.

Expressions for the equilibrium C_e and frozen C_f speeds of sound in a vapor–gas–liquid mixture can be obtained from the dispersion relation (4.1), (4.2) in the limit $\omega \to 0$ and $\omega \to \infty$, respectively:

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*Equations (4.1) and (4.2) reduce to the dependence of [2] when τ_β is replaced with τ_Σ and use is made of (3.3).

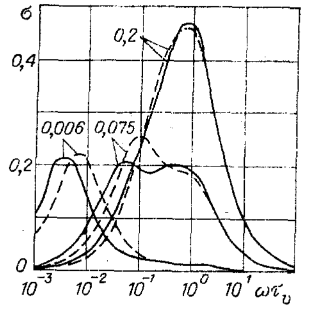

Fig. 1

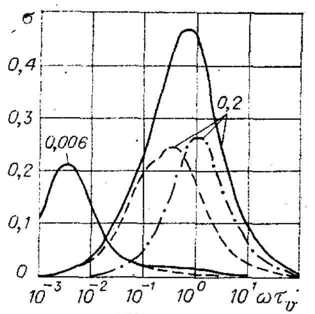

Fig. 2

$$C_e = C_1\left[\frac{\gamma_e}{(1+m)\gamma_1}\right]^{1/2}, \quad C_f = C_1,$$

$$\gamma_e = \frac{k_{V0} + (1 - rk_{V0})\,[1-(r+h)\,k_{V0}]\,(\bar{c}_1 + m\bar{c}_2)\,b/(\gamma_1 \bar{l}^2)}{k_{V0} + (1 - rk_{V0})\left[(1 - rk_{V0})\left(\dfrac{\bar{c}_1}{\gamma_1} + m\bar{c}_2\right)b - \left(b + \dfrac{h}{(1-rk_{V0})}\right)\bar{l}k_{V0} + \dfrac{hbk_{V0}}{\gamma_1}\right]\Big/(\gamma_1\bar{l}^2)}. \tag{4.4}$$

Expressions for C_e in a one-component mixture with phase transitions (vapor and liquid) and in a mixture without phase transitions (gas with solid particles) follow from (4.4) by taking the limits $k_{V0} \to 1$ and 0, respectively. In the case $m = 0$ and $k_{V0} = 0$, when the dispersed phase and the vapor component of the gaseous phase are both absent, $C_e = C_1$.

5. Results. Dispersion curves, calculated with the help of (4.1) and (4.2) for a mixture of air with water droplets and water vapor (water droplets of radius $a = 2 \cdot 10^{-6}$ m and pressure of the gaseous phase $p_{10} = 0.1$ MPa) are shown in Figs. 1 through 5. The curves of Figs. 1 through 3 correspond to a vapor concentration in the carrier phase of $k_{V0} = 0.8 \cdot 10^{-2}$ [$T_0 = T_S(k_{V0}) = 280$ K]. The numbers labelling the curves indicate the value of the mass content m of drops in the mixture.

Figure 1 shows the effect of the mass content of the drops on the attenuation decrement per wavelength as a function of the dimensionless frequency $\omega\tau_V$. The dashed and solid curves correspond to quasi-equilibrium ($\beta = \infty$, $\tau_\beta = 0$) and nonequilibrium ($\beta = 0.04$) mass exchange. It is evident that for a given vapor concentration, departure of the phase transition from equilibrium occurs when $\omega\tau_V \lesssim 1$.* The effect of departure of the mass exchange from equilibrium depends on m. For $m = 0.006$ it leads to a marked shift in the extremum of the curve $\sigma(\omega\tau_V)$ into the low-frequency region. For $m = 0.2$ the contribution of nonequilibrium effects is less important. For small mass content of the dispersed phase the maximum value of the coefficient σ depends weakly on m. For example, as m varies from 0.006 to 0.075 (i.e., by about an order of magnitude) there is practically no change in $\sigma_{\max}$.

A characteristic feature of one and two-component gas–liquid mixtures with phase transitions is that the dependence of the attenuation decrement σ on $\omega\tau_V$ can have two maxima (Fig. 1). One of them is due to dissipation because of interphase friction and occurs for $\omega\tau_V \sim 1$, while the other is observed for $\omega\tau_V \sim m$ and is due to the dissipative effects of nonequilibrium heat and mass exchange between the phases. We consider the individual contributions of interphase friction and heat and mass exchange to the dispersion and dissipation. The contributions of friction (dash-dotted curves) and heat and mass exchange (dashed curves) to the total attenuation coefficient σ, phase velocity C_p, and linear coefficient K_{**} (given by the solid curves) are shown in Figs. 2 and 3. It is evident that when the mass content of drops is small ($m = 0.006$) the dependence $\sigma(\omega\tau_V)$ is determined by heat and mass exchange between the phases (when $\omega\tau_V \lesssim 10^{-1}$ the dashed curves approach the solid curves, see Fig. 2). As m increases, the effect of friction on the dispersion and dissipation of the excitations increases (Figs. 2 and 3). However, there always exists a region of moderate values of m (whose size depends on k_{V0} and on the thermal and physical properties of the phases) for which the attenuation of low-frequency signals ($\omega\tau_V \ll 1$) is determined mainly by heat and mass exchange between the phases.

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*In correspondence with the estimate (4.3), the effects of nonequilibrium mass exchange must appear for any frequency. However from Fig. 1 it is seen that they are important only when $\omega\tau_V \lesssim 1$. This is because at higher frequencies the relative contribution of mass exchange to the dissipation is small in comparison with the contribution of interphase friction.

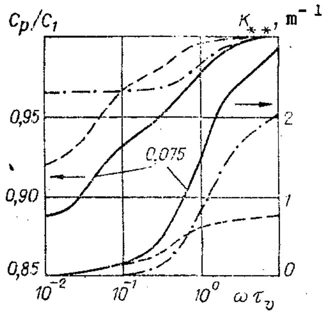

Fig. 3

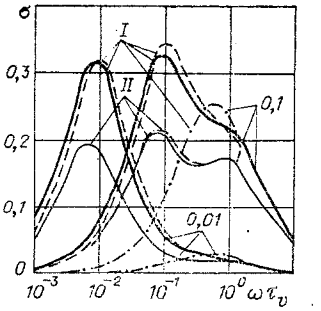

Fig. 4

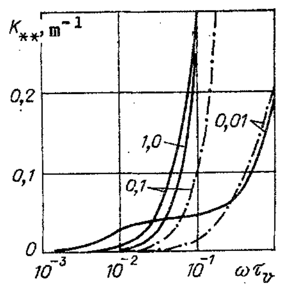

Fig. 5

Analysis of the interaction between the phases in vapor–gas–liquid mixtures with small particle mass contents ($m \ll 1$) shows that in addition to the characteristic time τ_V [Eq. (2.1)] there exists another characteristic time for heat and mass exchange between the phases

$$\tau_m = \frac{3}{2}\frac{\tau_v}{m}\left[\frac{\gamma_1 \bar{l}^2 k_{V0}\mathrm{Pr} + (1-k_{V0})\, b\bar{c}_1\,\mathrm{Sc} + \frac{2\bar{c}_1}{3r}\frac{\tau_\beta}{\tau_v}}{\gamma_1 \bar{l}^2 k_{V0} + (1-k_{V0})\, b\bar{c}_1}\right], \quad 0<m\ll 1. \tag{5.1}$$

The contribution of heat and mass exchange to the dispersion and dissipation of excitations begins to appear for $\omega\tau_m \sim 10^{-1}$, and the contribution of interphase friction for $\omega\tau_V \sim 10^{-1}$. Usually Pr, Sc ~ 1, then, according to (5.1), when $\tau_\beta = 0$ we have $\tau_m \sim \tau_V/m$, i.e., $\tau_m \gg \tau_V$. When $\omega\tau_m \sim 1$ the vapor–gas–liquid mixture is in equilibrium with respect to velocity ($v_1 \sim v_2$) but out of equilibrium both with respect to characteristic temperature ($T_1 \neq T_2 \neq T_\Sigma$) and with respect to vapor concentration in the vapor–gas mixture ($k_{V\Sigma} \neq k_V$). The important effect on the dispersion and dissipation of excitations at these frequencies is heat and mass exchange between the phases. As the frequency increases, the relative contribution of heat and mass exchange diminishes. When $\omega\tau_V \gtrsim 1$, the important contribution is interphase friction.

The effect of a departure from equilibrium of the phase transitions on the dependence $\sigma(\omega\tau_V)$ is shown in Fig. 4 for different values of k_{V0} and m. Curves I and II correspond to $k_{V0} = 0.1$ and 1, $T_0 = 327$ and 373°K. The dashed and solid curves correspond to equilibrium ($\beta = \infty$) and nonequilibrium ($\beta = 0.04$) phase transitions. The dash-dotted curves correspond to the case of frozen mass exchange [for $k_{V0} = 0.1$ ($T_0 = 327$°K)]. It is evident that the nonequilibrium nature of the mass exchange (measured by the difference between T_Σ and T_S) on the function $\sigma(\omega\tau_V)$ decreases as k_{V0} increases. In the case of a one-component medium ($k_{V0} = 1$, curve II), the effect of nonequilibrium mass exchange on the function $\sigma(\omega\tau_V)$ is weaker. We note that when $m \sim 0.01$ the maximum value of the coefficient σ is roughly the same for $k_{V0} = 1$ and $0.8 \cdot 10^{-2}$: $\sigma_{\max} \sim 0.2$ (see Fig. 1). This is because when $m = 0.01$ the curve $\sigma(k_{V0})$ has an extremum $\sigma_{\max} \sim 0.3$ corresponding to $k_{V0} \sim 0.1$ ($T_0 = 327$°K).

The typical dependence of K_{**} on $\omega\tau_V$ is shown in Fig. 5 for $k_{V0} = 0.1$. The solid curves are for $\beta = 0.04$ and the dash-dotted curves correspond to $\beta = 0$. It is evident that when $\omega\tau_V \sim 10^{-3}$-$10^{-2}$, excitations in a medium with $m = 10^{-2}$ are attenuated much more strongly than in a mixture with a significantly higher mass content of the dispersed phase ($m \sim 0.1$-1). A more careful analysis shows that in the presence of phase transitions the attenuation coefficient for low-frequency excitations ($\omega\tau_V \ll 1$) depends on m in a nonmonotonic way. For example, $K_{**}(m)$ has a maximum for $m = 10^{-2}$ when $\omega\tau_V \sim 10^{-3}$-$10^{-2}$ ($m = 0$, $K_{**} = 0$; $m = 1$, $K_{**} \sim 0$-0.005). When $\omega\tau_V \sim 10^{-2}$ the attenuation in a suspension with phase transitions can be several orders of magnitude larger than that in a gas with solid particles, where phase transitions are absent (Fig. 5).

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PULSATION SPECTRUM OF THE MIXING LAYER OF AN UNDEREXPANDED JET

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UDC 533.6.011

The very numerous investigations of the gas dynamics of underexpanded jets that have been made up to now have made it possible to find the determining parameters and the main laws of outflow [1-3]. The character of flow in the mixing layer of the initial section of a supersonic underexpanded jet is determined by the Reynolds number Re_L calculated from the distance to the central compression shock, the maximum outflow velocity, and the parameters of the flooded space [1]. A turbulent flow regime is observed for $\mathrm{Re}_L > 10^4$. Because of the fact that the velocity drop over the thickness of the mixing layer has the order of magnitude of the speed of sound, turbulent gas mixing can lead to considerable pulsations of the gas-dynamic parameters. A study of the fluctuation quantities in such flows is associated with a number of difficulties. At the experimental level the problem consists in the necessity of using diagnostic methods with high temporal and spatial resolution.

In the present paper we investigate density pulsations in the initial section of a supersonic underexpanded jet escaping from a sonic nozzle. It proved possible to formulate this work in connection with the creation of a pulsed local method of density measurement, based on Rayleigh scattering of light [4].

Diagnostic Method and Experimental Setup

The use of the method of Rayleigh scattering to measure the concentrations of molecules in gas streams has a number of advantages over other methods [5]: the noncontact nature and the high localization of the measurements. But the drawbacks limit its wide application. First, the scattering cross section is rather small, and the traditional use of continuous lasers as the radiation source requires the use of storage systems to isolate the signal against the noise background. Therefore, investigations with a high time resolution are impossible. Second, the cross section for scattering on dust particles is proportional to the sixth power of their size, so that the use of the method in actual flows is hindered (in air under standard conditions, for example, the total number of dust particles is 10^4-10^5 cm^{-3} [6]).

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